Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Friday 13 October 2023

Afternoon (Time: 1 hour 30 minutes) | Paper reference **WMA12/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Pure Mathematics P2

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

1. Given that a, b and c are integers greater than 0 such that

- $c = 3a + 1$
- $a + b + c = 15$

prove, by exhaustion, that the product abc is always a multiple of 4

You may use the table below to illustrate your answer.

(3)

You may not need to use all rows of this table.

a	b	c	abc
1	10	4	40
2	6	7	84
3	2	10	60
4	−2	13	×

↳ does not satisfy conditions.

↳ Given :

$a + b + c = 15 \longrightarrow b = 15 - a - c$

$c = 3a + 1$

We can use the two equations to find all possible values of a, b, c

(1) let $a = 1$:

$c = 3(1) + 1 = 4$

$b = 15 - 4 - 1 = 10$

(2) let $a = 2$

$c = 3(2) + 1 = 7$

$b = 15 - 7 - 2 = 6$

(3) let $a = 3$

$c = 3(3) + 1 = 10$

$b = 15 - 10 - 3 = 2$

→ note : If you try with $a \geq 4$, b will be negative, which doesn't satisfy the conditions provided in the question.

Question 1 continued

So:

when $a = 1$
$b = 10$
$c = 4$, $abc = 40 = 4(10)$

when $a = 2$
$b = 6$
$c = 7$, $abc = 84 = 4(21)$

when $a = 3$
$b = 2$
$c = 10$, $abc = 60 = 4(15)$

$\therefore$ All values of abc are multiples of 4.

(Total for Question 1 is 3 marks)

2. A sequence $u_1, u_2, u_3, \dots$ is defined by

Starting point

$$u_1 = 3$$

$$\boxed{u_{n+1} = 2 - \frac{4}{u_n}}$$

recursive sequence (plugging in a term on the right gives the next term on the left)

(a) Find the value of u_2, the value of u_3 and the value of u_4

(3)

(b) Find the value of

end

$$\sum_{r=1}^{100} u_r$$

terms : $u_1 + u_2 + u_3 \dots$

sum

starting value

(2)

(a) This is a recursive sequence :

$$u_{n+1} = 2 - \frac{4}{u_n}$$

→ notice how $n+1$ is 1 more than n

Let's find :

① u_2 :

$$u_2 = 2 - \frac{4}{u_1} = 2 - \frac{4}{3} = \frac{2}{3}$$

② u_3 :

$$u_3 = 2 - \frac{4}{u_2} = 2 - \frac{4}{2/3} = 2 - \frac{4(3)}{2} = -4$$

③ u_4 :

$$u_4 = 2 - \frac{4}{u_3} = 2 - \frac{4}{-4} = 3$$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 2 continued

(b) $\sum_{r=1}^{100} u_r$ means start the sequence at $r=1$ and sum all the terms up to $n = 100$

$$\sum_{r=1}^{100} u_r = u_1 + u_2 + u_3 + u_4 + \dots + u_{100}$$

We know:

$u_1 = 3$
$u_2 = 2/3$
$u_3 = -4$
$u_4 = 3$

same value

Therefore, the series is periodic and values repeat themselves every 3 terms.

$$\underbrace{3 + \frac{2}{3} + -4}+\underbrace{3 + \frac{2}{3} + -4} + \dots$$

so: $\dfrac{100}{3 \text{ terms}} = 33 + 1$ → remainder; one more term.

33 repeats

$$\sum_{r=1}^{100} u_r = 33\left(3 + \frac{2}{3} + (-4)\right) + 3 = -8$$

(Total for Question 2 is 5 marks)

3. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Solve, for $0 < \theta \leqslant 360°$ the equation

$$2\tan\theta + 3\sin\theta = 0$$

giving your answers, as appropriate, to one decimal place.

(5)

(b) Hence, or otherwise, find the smallest positive solution of

$$2\tan(2x + 40°) + 3\sin(2x + 40°) = 0$$

giving your answer to one decimal place.

(2)

(a) $2\tan\theta + 3\sin\theta = 0 \rightarrow \tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$\left[2\dfrac{\sin\theta}{\cos\theta} + 3\sin\theta = 0 \right] \times \cos\theta$

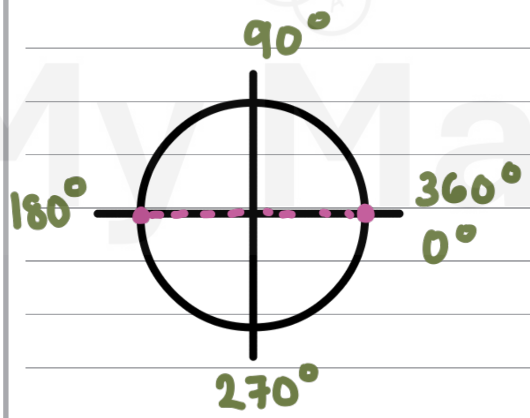
$2\sin\theta + 3\sin\theta\cos\theta = 0$ (take common factor)

$\sin\theta(2 + 3\cos\theta) = 0$

$\sin\theta = 0$ or $2 + 3\cos\theta = 0$; $0° \leq \theta < 360°$

① $\sin\theta = 0$

ON UNIT CIRCLE:



$\sin\theta = 0$ at 3 points:

$\theta = 0$

$\theta = 180°$

$\theta = 360°$ × not included in the given range.

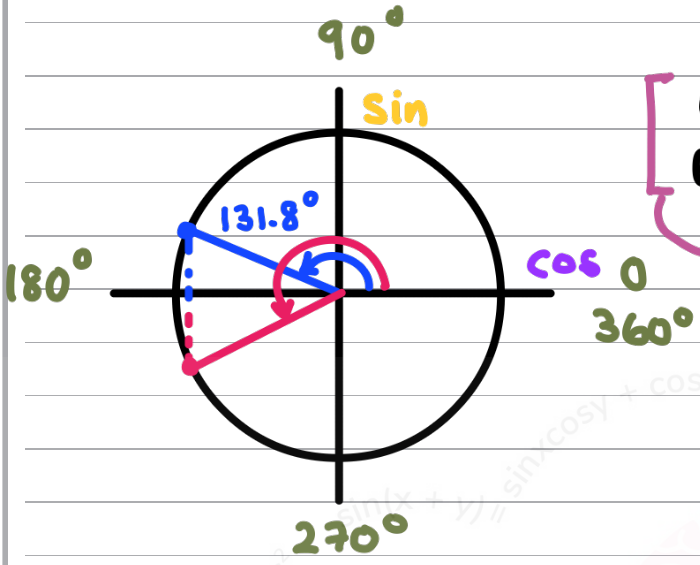
② $2 + 3\cos\theta = 0$

$\cos\theta = \dfrac{-2}{3}$

$\theta = \arccos\left(\dfrac{-2}{3}\right) = 131.8°$

Question 3 continued

ON UNIT CIRCLE :



$\theta = 131.8°$

$\theta = 360° - 131.8° = 228.2°$

both have same values for $\cos\theta$

so both are solutions

so : $\theta = 0,\ 131.8°,\ 228.2°,\ 360°$

(b) $2\tan(2x + 40°) + 3\sin(2x + 40) = 0$

$2\tan(\theta) + 3\sin(\theta) = 0$

so : $\theta = 2x + 40°$

$0 = 2x + 40 \longrightarrow x = \dfrac{0 - 40}{2} = -20$ ✗ not +ve

$131.8 = 2x + 40 \longrightarrow x = \dfrac{131.8 - 40}{2} = 45.9°$

$228.2 = 2x + 40 \longrightarrow x = \dfrac{228.2 - 40}{2} = 94.1°$

$360° = 2x + 40 \longrightarrow x = \dfrac{360 - 40}{2} = 80°$

smallest value = $45.9°$

(Total for Question 3 is 7 marks)

4. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

$$f(x) = 4x^3 + ax^2 - 29x + b$$

where a and b are constants.

Given that $(2x + 1)$ is a factor of $f(x)$,

(a) show that

$$a + 4b = -56$$

(2)

Given also that when $f(x)$ is divided by $(x - 2)$ the remainder is -25

(b) find a second simplified equation linking a and b.

(2)

(c) Hence, using algebra and showing your working,

(i) find the value of a and the value of b,

(ii) fully factorise $f(x)$.

(5)

(a) 1 Set the factor equal to 0 :

$2x + 1 = 0$

$x = -\frac{1}{2}$

2 Subtitute $x = -1/2$ into $f(x) = 0$:

$4x^3 + ax^2 - 29x + b = 0$

$4\left(-\frac{1}{2}\right)^3 + a\left(-\frac{1}{2}\right)^2 - 29\left(-\frac{1}{2}\right) + b = 0$

$-\frac{1}{2} + \frac{1}{4}a + \frac{29}{2} + b = 0$

$\left[\frac{1}{4}a + b = -14\right] \times 4$

$a + 4b = 56$

(b) USE : REMAINDER THEOREM :

if $\frac{f(x)}{(x-k)}$ = quotient + remainder :

then $f(k)$ = remainder

P 7 4 3 1 9 A 0 8 3 2

Question 4 continued

so: if $(x-2)$ gives remainder $= -25$
then $f(2) = -25$

substitute $x=2$ into $f(x) = -25$:

$$4x^3 + ax^2 - 29x + b = -25$$
$$4(2)^3 + a(2)^2 - 29(2) + b = -25$$
$$32 + 4a - 58 + b = -25$$
$$4a + b = 1$$

(c)(i) SOLVE SIMULTANEOUSLY :

METHOD 1 : elimination

$4a + b = 1$
$(a + 4b = -56) \times 4$

$\ominus$ $\cancel{4a} + b = 1$
$\cancel{4a} + 16b = -224$
$-15b = 225$
$b = -15$

$a = -56 - 4(-15) = 4$

METHOD 2 : substitution

$4a + b = 1 \rightarrow b = 1 - 4a$
$a + 4b = -56$

$a + 4(1-4a) = -56$
$a + 4 - 16a = -56$
$-15a = -60$
$a = 4$

$b = 1 - 4(4) = -15$

(ii) $f(x) = 4x^3 + 4x^2 - 29x - 15$
use algebraic division to factorise :

$2x+1 \enclose{longdiv}{4x^3 + 4x^2 - 29x - 15}$

(1) Divide first term of dividend by first term of divisor : $\frac{4x^2}{2x} = 2x^2 \rightarrow$ first term of quotient

$$\begin{array}{r} 2x^2 \\ 2x+1 \enclose{longdiv}{4x^3 + 4x^2 - 29x - 15} \end{array}$$

$\div$

P 7 4 3 1 9 A 0 9 3 2

Question 4 continued

(2) Multiply out and write under :

$$\begin{array}{r|l} & 2x^2 \\ 2x+1 & 4x^3+4x^2-29x-15 \\ & 4x^3+2x^2 \\ \hline \end{array}$$

(3) Subtract then bring down next term :

$$\begin{array}{r|l} & 2x^2 \\ 2x+1 & 4x^3+4x^2-29x-15 \\ & -(4x^3+2x^2) \downarrow \\ \hline & \quad 2x^2-29x \end{array}$$

(4) Divide first term of expression by first term of divisor : $\frac{2x^2}{2x} = x \rightarrow$ next term of quotient

$$\begin{array}{r|l} & 2x^2+x \\ 2x+1 & 4x^3+4x^2-29x-15 \\ & -(4x^3+2x^2) \\ \hline & \quad 2x^2-29x \end{array}$$

(5) Multiply out and write under :

$$\begin{array}{r|l} & 2x^2+x \\ 2x+1 & 4x^3+4x^2-29x-15 \\ & -(4x^3+2x^2) \\ \hline & \quad 2x^2-29x \\ & \quad 2x^2+x \\ \hline \end{array}$$

(6) Subtract then bring down next term :

$$\begin{array}{r|l} & 2x^2+x \\ 2x+1 & 4x^3+4x^2-29x-15 \\ & -(4x^3+2x^2) \\ \hline & \quad 2x^2-29x \\ & \quad -(2x^2+x) \downarrow \\ \hline & \qquad -30x-15 \end{array}$$

Question 4 continued

(7) Divide first term of expression by first term of divisor : $\frac{-30x}{2x} = -15 \rightarrow$ next term of quotient

$$
\begin{array}{r|l}
 & 2x^2 + x - 15 \\
\hline
2x+1 & 4x^3 + 4x^2 - 29x - 15 \\
 & -(4x^3 + 2x^2) \\
\hline
 & \quad 2x^2 - 29x \\
 & \quad -(2x^2 + x) \\
\hline
 & \qquad -30x - 15
\end{array}
$$

(8) Multiply out and write under :

$$
\begin{array}{r|l}
 & 2x^2 + x - 15 \\
\hline
2x+1 & 4x^3 + 4x^2 - 29x - 15 \\
 & -(4x^3 + 2x^2) \\
\hline
 & \quad 2x^2 - 29x \\
 & \quad -(2x^2 + x) \\
\hline
 & \qquad -30x - 15 \\
 & \qquad -30x - 15
\end{array}
$$

(9) Subtract

$$
\begin{array}{r|l}
 & 2x^2 + x - 15 \\
\hline
2x+1 & 4x^3 + 4x^2 - 29x - 15 \\
 & -(4x^3 + 2x^2) \\
\hline
 & \quad 2x^2 - 29x \\
 & \quad -(2x^2 + x) \\
\hline
 & \qquad -30x - 15 \\
 & \qquad -(-30x - 15) \\
\hline
 & \qquad\qquad 0
\end{array}
$$

so : $4x^2 + 4x - 29x - 15 = (2x+1)(2x^2 + x - 15)$

$= (2x+1)(2x-5)(x+3)$

$f(x) = (2x+1)(2x-5)(x+3)$

(Total for Question 4 is 9 marks)

5. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(i) Solve

$$3^a = 70$$

giving the answer to 3 decimal places.

(2)

(ii) Find the exact value of b such that

$$4 + 3\log_3 b = \log_3 5b$$

(4)

(i) Use the rule :

$$\boxed{\log x^a = a\log x}$$

$\log 3^a = \log 70$ (take log of both sides).

$$\frac{a\cancel{\log 3}}{\cancel{\log 3}} = \frac{\log 70}{\log 3}$$

$$a = \frac{\log 70}{\log 3} = 3.867 \text{ (3 d.p.)}$$

(ii) $4 + 3\log_3 b = \log_3 5b$

$$4 = \log_3 5b - 3\log_3 b$$

Recall rules of logarithms :

$\log_a(b) + \log_a(c) = \log_a(b \times c)$

$\log_a(b) - \log_a(c) = \log_a\left(\frac{b}{c}\right)$

$$\log_3 5b - \log_3 b^3 = 4$$

RECALL!

$$\log_3\left(\frac{5b}{b^3}\right) = 4$$

$\log_a x = b$

$a^b = x$

P 7 4 3 1 9 A 0 1 2 3 2

DO NOT WRITE IN THIS AREA

Question 5 continued

$\frac{5\cancel{b}}{b^{2}} = 3^4$

$\frac{5}{b^2} = 81 \rightarrow b = \pm\sqrt{\frac{5}{81}}$

log of a negative is not possible !

$b = \sqrt{\frac{5}{81}}$ or $\frac{\sqrt{5}}{9}$

ALTERNATIVE SOLUTION :

If $4^3 = 81$

then $\log_3 81 = 4$

SO : $4 + 3\log_3 b = \log_3 5b$

$\log_3 81 + 3\log_3 b = \log_3 5b$

$\log_3 81 + \log_3 b^3 - \log_3 5b = 0$

$\log_3\left(\frac{81 \times b^3}{5b}\right) = 0$

$\frac{81 b^{\cancel{3}\,2}}{5\cancel{b}} = 3^0$

$\frac{81 b^2}{5} = 1$

$\sqrt{b^2} = \sqrt{\frac{5}{81}} \longrightarrow b = \sqrt{\frac{5}{81}}$

(Total for Question 5 is 6 marks)

6.

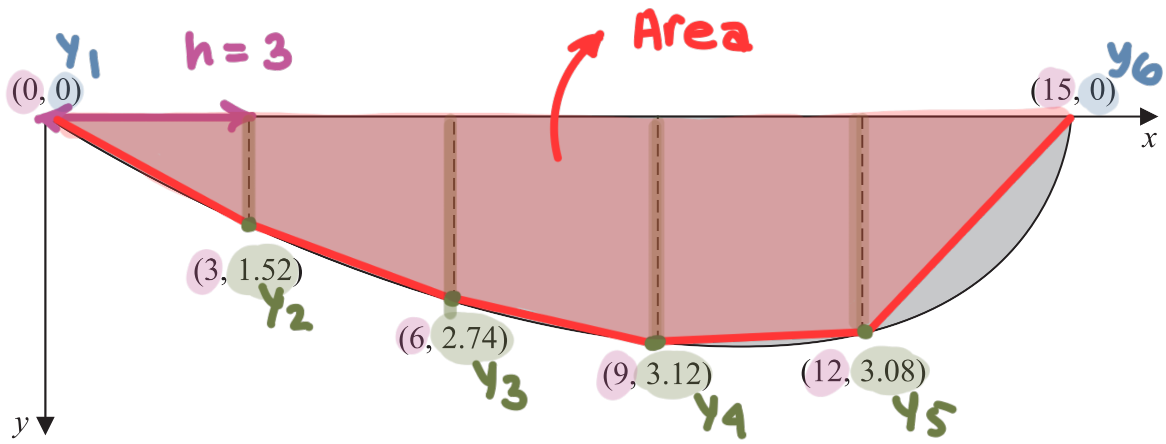


Figure 1

A river is being studied.

At one particular place, the river is 15 m wide.

The depth, y metres, of the river is measured at a point x metres from one side of the river.

Figure 1 shows a plot of the cross-section of the river and the coordinate values (x, y)

(a) Use the trapezium rule with all the y values given in Figure 1 to estimate the cross-sectional area of the river.

$\frac{\text{distance}}{\text{time}}$

(3)

The water in the river is modelled as flowing at a constant speed of $1.5\,\text{m s}^{-1}$ across the whole of the cross-section.

(b) Use the model and the answer to part (a) to estimate the volume of water flowing through this section of the river each minute, giving your answer in m^3 to 2 significant figures.

60 s

(2)

Assuming the model,

(c) state, giving a reason for your answer, whether your answer for part (b) is an overestimate or an underestimate of the true volume of water flowing through this section of the river each minute.

(1)

$A = \frac{1}{2}h\left[y_1 + 2(y_2 + y_3 + y_4 + y_5) + y_6\right]$

$A = \frac{1}{2}(3)\left[0 + 2(1.52 + 2.74 + 3.12 + 3.08) + 0\right]$

$A = \frac{3}{2}(20.92) = 31.38\ \text{m}^2 \approx 31.4\ (3\ \text{sf})$

Question 6 continued

(b) volume in one min $= \dfrac{\text{area} \times \text{distance}}{\text{time (s)}} \times 60\text{s}$

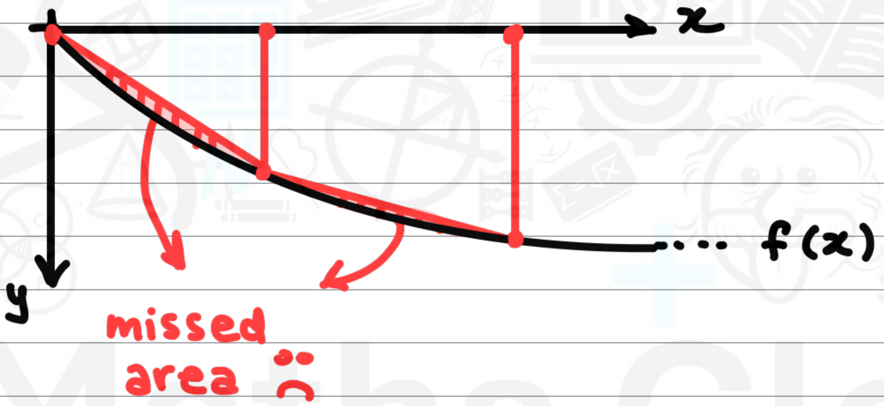
$= \text{area} \times \text{speed} \times 60$

$= 31.38 \times 1.5 \times 60 = 2824.2$

$= 2800 \text{ m}^3$ (2 s.f)

(c) It is an underestimate, since the CALCULATED cross-sectional area (using trapezium rule) is less than the actual cross-sectional area.

This is because the area of the trapezia are smaller than the cross-sectional area.



(Total for Question 6 is 6 marks)

7.

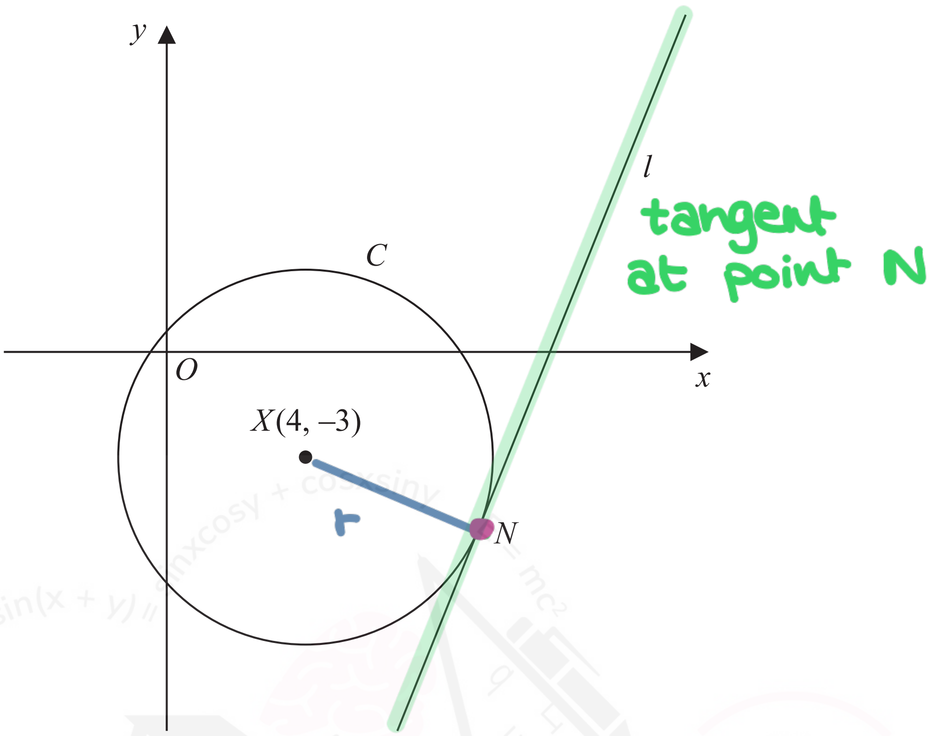


Figure 2

Figure 2 shows a sketch of

- the circle C with centre $X(4, -3)$
- the line l with equation $y = \frac{5}{2}x - \frac{55}{2}$

m_l

Given that l is the tangent to C at the point N,

(a) show that an equation for the straight line passing through X and N is

$$2x + 5y + 7 = 0$$

(3)

(b) Hence find

(i) the coordinates of N,

(ii) an equation for C.

(5)

(a) equation → $y = mx + c$

line XN is a radius of C, and line L is a tangent at point N. Therefore, line XN and line L are perpendicular according to circle theorems.

So : $m_{XN} = -\frac{1}{m_l}$

Question 7 continued

$m_{XN} = \dfrac{-1}{(5/2)} = \dfrac{-2}{5}$

$y = \dfrac{-2}{5}x + c$

Plug in $(4, -3)$ to find c

$-3 = \dfrac{-2}{5}(4) + c \longrightarrow c = \dfrac{-7}{5}$

$y = \dfrac{-2}{5}x - \dfrac{7}{5}$

The q asked for the answer in the form:

$\rightarrow ax + by + c$ so:

$5 \times (y) = \left(\dfrac{-2}{5}x - \dfrac{7}{5}\right) \times 5$

$5y = -2x - 7$

$2x + 5y + 7 = 0$

(b) (i) Point N is shared by line XN and line l. So:

$y_{\text{line } XN} = y_{\text{line } l}$ (at point N)

$\dfrac{-2}{5}x - \dfrac{7}{5} = \dfrac{5}{2}x - \dfrac{55}{2}$

$\dfrac{29}{\cancel{10}}x = \dfrac{261}{\cancel{10}}$

$x = 9$

$y_N = \dfrac{5}{2}(9) - \dfrac{55}{2} = -5$

P 7 4 3 1 9 A 0 1 7 3 2

Question 7 continued

so : $\text{coords}_N = (9, -5)$

(ii) Eq. of circle :

$$(x - x_{\text{centre}})^2 + (y - y_{\text{centre}})^2 = r^2$$

FIND R :

$$(9 - 4)^2 + (-5 - (-3))^2 = r^2$$

$$25 + 4 = r^2$$

$$29 = r^2$$

equation : $(x - 4)^2 + (y + 3)^2 = 29$

Question 7 continued

(Total for Question 7 is 8 marks)

8. In a large theatre there are n rows of seats, where n is a constant.

The number of seats in the first row is a, where a is a constant. term 1 (a_1)

In each subsequent row there are 4 more seats than in the previous row so that

- in the 2nd row there are $(a + 4)$ seats term 2 (a_2)
- in the 3rd row there are $(a + 8)$ seats term 3 (a_3)
- the number of seats in each row form an **arithmetic** sequence

Given that the **total** number of seats in the first 10 rows is 360 sum

→ sum up to $a_{10} = 360$

(a) find the value of a.

(2)

Given also that the total number of seats in the n rows is 2146 sum

→ sum up to $a_n = 2146$

(b) show that

$$n^2 + 8n - 1073 = 0$$

(2)

(c) Hence

(i) state the number of rows of seats in the theatre,

(ii) find the maximum number of seats in any one row.

(3)

(a) $a_1 + a_2 + a_3 + \dots + a_{10} = 360$

use partial sum formula:

$$S_n = \frac{n}{2}(2a_1 + (n-1)d)$$

$$S_{10} = \frac{10}{2}(2a + (10-1)d) = 360$$

Find d: $a + 4 - a = 4$

$$S_{10} = \frac{10}{2}(2a + 9(4)) = 360$$

$$\frac{1}{5} \times (5(2a + 36)) = (360) \times \frac{1}{5}$$

$$2a + 36 = 72$$

$$a = 18$$

DO NOT WRITE IN THIS AREA

Question 8 continued

(b) $S_n = \frac{n}{2}\left(2a_1 + (n-1)d\right)$

$S_n = \frac{n}{2}\left(2a + (n-1)(4)\right) = 2146$

$\frac{n}{2}\left(2(18) + 4n - 4\right) = 2146$

$\frac{\cancel{2}(18)n}{\cancel{2}} + \frac{4n^2}{2} - \frac{4n}{2} = 2146$

$18n + 2n^2 - 2n = 2146$

$\frac{1}{2} \times (2n^2 + 16n - 2146) = 0 \times \frac{1}{2}$

$n^2 + 8n - 1073 = 0$

(c) (i) Solve for n:

$n^2 + 8n - 1073 = 0$

$(n - 29)(n + 37)$

$n = 29$

$n = -37$ x reject

so $n = 29$

(ii) $18 \xrightarrow{+4} 22 \xrightarrow{+4} 26 \xrightarrow{+4} \ldots$

we know no. of seats is increasing, so max no. of seats is in the last row, $n=29$

$a_n = a_1 + (n-1)d$

$a_{29} = 18 + (29-1)(4) = 130$ seats

Question 8 continued

Question 8 continued

(Total for Question 8 is 7 marks)

9.

$A = \int_0^9 \left(\frac{2}{3}x^2 - 9\sqrt{x} + 13\right) dx$

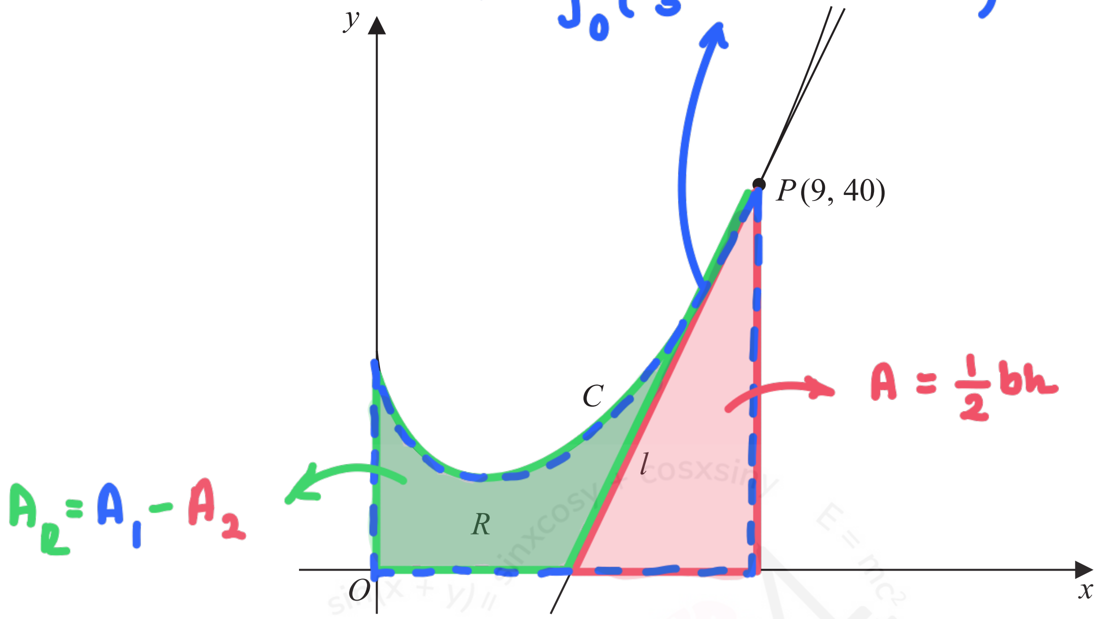


Figure 3

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Figure 3 shows a sketch of part of the curve C with equation

$$y = \frac{2}{3}x^2 - 9\sqrt{x} + 13 \qquad x \geqslant 0$$

(a) Find, using calculus, the range of values of x for which y is increasing. **(4)**

$\frac{dy}{dx} > 0$

The point P lies on C and has coordinates $(9, 40)$.

The line l is the tangent to C at the point P.

The finite region R, shown shaded in Figure 3, is bounded by the curve C, the line l, the x-axis and the y-axis.

(b) Find, using calculus, the exact area of R. **(8)**

NOTE !

(a) When $\frac{dy}{dx} < 0$, y is decreasing

When $\frac{dy}{dx} = 0$, y is stationary

When $\frac{dy}{dx} > 0$, y is increasing

① Find dy/dx :

DO NOT WRITE IN THIS AREA

Question 9 continued

$$y = \frac{2}{3}x^2 - 9x^{1/2} + 13$$

DIFFERENTIATION

$$y = ax^n$$
$$\frac{dy}{dx} = nax^{n-1}$$

$$\frac{dy}{dx} = (2)\frac{2}{3}x^{2-1} - \left(\frac{1}{2}\right)9x^{1/2-1}$$

$$\frac{dy}{dx} = \frac{4}{3}x - \frac{9}{2}x^{-1/2}$$

② let $\frac{dy}{dx} > 0$: $\frac{4}{3}x - \frac{9}{2\sqrt{x}} > 0$

$$6\sqrt{x} \times \left(\frac{4}{3}x - \frac{9}{2\sqrt{x}}\right) > 0 \times 6\sqrt{x}$$

$$8x^{3/2} - 27 > 0$$

$$\frac{1}{8} \times (8x^{3/2}) > 27 \times \frac{1}{8}$$

$$\left(x^{3/2}\right)^2 > \left(\frac{27}{8}\right)^2$$

$$\sqrt[3]{x^3} > \sqrt[3]{\frac{729}{64}}$$

$$x > \frac{9}{4}$$

(b) $A_R = \int_0^9 \frac{2}{3}x^2 - 9x^{1/2} + 13 \, dx - A_{\text{under } \ell}$

① Find $\int_0^9 \frac{2}{3}x^2 - 9x^{1/2} + 13 \, dx$

P 7 4 3 1 9 A 0 2 5 3 2

Question 9 continued

REMEMBER!

$$\boxed{\int x^n\,dx = \frac{x^{n+1}}{n+1}}$$

$$\int_0^9 \frac{2}{3}x^2 - 9x^{1/2} + 13 \; dx$$

$$= \left[\frac{2}{3(2+1)}x^{(2+1)} - \frac{9}{(1/2+1)}x^{(1/2+1)} + \frac{13}{(0+1)}x^{0+1}\right]_0^9$$

$$= \left[\frac{2}{9}x^3 - 6x^{3/2} + 13x\right]_0^9$$

$$= \left(\frac{2}{9}(9)^3 - 6(9)^{3/2} + 13(9)\right) - 0$$

$$= 162 - 162 + 117 = 117$$

(2) Find A under ℓ

- First, find length of base :

$= x_P -$ x-intercept of l

eq. of line ℓ : $y = mx + c$

$m = \frac{dy}{dx}$ at point P :

$$\frac{dy}{dx} = \frac{4}{3}(9) - \frac{9}{2}(9)^{-1/2} = \frac{21}{2}$$

$$0 - 40 = \frac{21}{2}(x - 9)$$

$$-40 = \frac{21}{2}x - \frac{189}{2} \rightarrow x = \frac{109}{21}$$

DO NOT WRITE IN THIS AREA

Question 9 continued

So : length of base = $9 - \frac{109}{21} = \frac{80}{21}$

height = $y_p = 40$

so $A_{\text{under } \ell} = \frac{1}{2}\left(\frac{80}{21}\right)(40) = \frac{1600}{21}$

so $A_R = 117 - \frac{1600}{21} = \frac{857}{21}$

(Total for Question 9 is 12 marks)

10. (i) (a) Find, in ascending powers of x, the 2nd, 3rd and 5th terms of the binomial expansion of

$$(3 + 2x)^6$$

(3)

For a particular value of x, these three terms form consecutive terms in a geometric series.

(b) Find this value of x.

(3)

(ii) In a **different** geometric series,

- the first term is $\sin^2\theta$ $\quad a_1 = \sin^2\theta$
- the common ratio is $2\cos\theta$ $\quad r = 2\cos\theta$
- the sum to infinity is $\frac{8}{5}$ $\quad S_\infty = \frac{8}{5}$

(a) Show that

$$5\cos^2\theta - 16\cos\theta + 3 = 0$$

(3)

(b) Hence find the exact value of the 2nd term in the series.

(3)

(i)(a) Binomial expansion formula :

$$\boxed{(a+bx)^n = {}^nC_0\, a^n (bx)^0 + {}^nC_1\, a^{n-1}(bx)^1 + {}^nC_2\, a^{n-2}(bx)^2 + \dots}$$

$${}^nC_r = \frac{n!}{(n-r)!\,r!}$$

n : power of binomial

r : (position of term) -1

$\boxed{{}^nC_1 \text{ is ALWAYS } = n}$

→ 2nd term $= {}^6C_1 (3)^{6-1}(2x)^1$

$= 6(243)(2)x = 2916x$

→ 3rd term $= {}^6C_2 (3)^{6-2}(2x)^2$

$= \frac{6!}{(6-2)!\,2!}(81)(4)x^2 = 4860x^2$

→ 5th term $= {}^6C_4 (3)^{6-4}(2x)^4$

DO NOT WRITE IN THIS AREA

Question 10 continued

$$= \frac{6!}{(6-4)!\,4!}(9)(16)x^4 = 2160x^4$$

(b) In a geometric sequence :

$$\underset{a_1}{2916x}, \quad \underset{a_2}{4860x^2}, \quad \underset{a_3}{2160x^4}$$

Common ratio $(r) = \dfrac{a_n}{a_{n-1}}$

$$r = \frac{a_2}{a_1} = \frac{a_3}{a_2}$$

$$\frac{4860x^{\cancel{2}\,1}}{2916\cancel{x}} = \frac{2160x^{\cancel{4}\,2}}{4860\cancel{x^2}}$$

$$\frac{9}{4x} \times \left(\frac{5}{3}x\right) = \left(\frac{4}{9}x^2\right) \times \frac{9}{4x}$$

$$x = \frac{15}{4}$$

(ii) (a) Use sum to infinity formula :

↳ S_∞ converges ONLY if : $-1 < r < 1$

where $S_\infty = \dfrac{a}{1-r}$

$$\frac{8}{5} = \frac{\sin^2\theta}{1-2\cos\theta}$$

$$\boxed{\text{Use identity : } \sin^2\theta + \cos^2\theta = 1 \;\; \Rightarrow \;\; \sin^2\theta = 1 - \cos^2\theta}$$

Question 10 continued

$$\frac{8}{5} \cancel{=} \frac{1-\cos^2\theta}{1-2\cos\theta}$$

$$8(1-2\cos\theta) = 5(1-\cos^2\theta)$$
$$8-16\cos\theta = 5-5\cos^2\theta$$

$$5\cos^2\theta - 16\cos\theta + 3 = 0$$

(b) $5\cos^2\theta - 16\cos\theta + 3 = 0$

FACTORISE: $(5\cos\theta - 1)(\cos\theta - 3) = 0$

$\cos\theta = \frac{1}{5}$ or $\cos\theta = 3$

reject as range of $y=\cos\theta$ is $-1 \leq y \leq 1$

second term $= ar^{(2-1)}$
$= ar$
$= \sin^2\theta \times 2\cos\theta$
$= (1-\cos^2\theta) \times 2\cos\theta$
$= \left(1-\left(\frac{1}{5}\right)^2\right) \times 2\left(\frac{1}{5}\right) = \frac{48}{125}$

Question 10 continued

Question 10 continued

(Total for Question 10 is 12 marks)

TOTAL FOR PAPER IS 75 MARKS